High performance modeling of the transport of energetic particles for radiotherapy.

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Outline

- Introduction
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- Electron and photon validation
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Introduction

Overview

- In last decades new technologies in radiotherapy facilities have seen a fast development in order to damage more precisely the target tumor;

- The use of new sources of energetic particles and photons, time modulated beams with controlled spatial profiles and intensity represents a new generation of high-precision radiotherapy;

- In this direction it is important that the algorithms used in clinical practice become as precise and fast in dose distribution calculations as the facilities can be;

- Monte Carlo method is a possible solution to attend the requested accuracy but is too much time consuming;

- A deterministic algorithm, Acuros®, for external photon beam treatment planning has been proposed recently but, due to the discrete ordinates method used to discretize in angle, the accuracy or the speed could be reduced.
Introduction
The problem

- The dose calculation algorithms used in clinical practice are:

  - **Pencil beam**: really fast but inaccurate in case of complex geometry;

  - **Collapsed cone**: widely used in commercial TPS are a good compromise between accuracy and speed. But differences (greater than 5%) at the interfaces of materials (lung and tissue).

  - **Monte Carlo**: is equivalent to measurements in term of reliability but it suffers of time consuming calculation.

  - **Deterministic (Acuros®)**: this algorithm is already commercially proposed but theoretically should be slower than our method.

GX Ding et al., Int J Radiat Oncol Biol Phys. 2005
Introduction
The POPRA project

The aim of the ‘Programme Optique Physique et Radiothérapie en Aquitaine’ (POPRA) is to provide an alternative tool to standard models that allows the full employment of new technologies (as J. Page talk has shown);

For that our program is devoted both in theoretical and experimental studies:

- Modelling: the M1 algorithm with its entropic closure
- Validation: comparisons both with Monte Carlo and measurements in clinic
M_1 deterministic model

The linear Boltzmann transport equation

The deterministic method is based on the 3 dimensional linear Boltzmann transport equation (LBTE), calculating the particle distribution function:

$$\Omega \cdot \nabla \psi(r, \epsilon, \Omega) = \rho_{in}(r) \int_{\epsilon}^{\infty} \int_{S^2} \sigma_{in}(\epsilon', \epsilon, \Omega, \Omega') \psi(r, \epsilon', \Omega') d\Omega' d\epsilon' + \rho_{el}(r) \int_{S^2} \sigma_{el}(r, \epsilon, \Omega, \Omega') \psi(r, \epsilon, \Omega') d\Omega' - \rho_{tot}(r) \sigma_{tot}(\epsilon) \psi(r, \epsilon, \Omega).$$

To be numerically solved, the LBTE has to be discretized.

The LBT equation has 6 variables. To be numerically solved and to reduce the computational time approximations are needed.
The linearized Boltzmann transport equation is:

\[
\Omega \cdot \nabla \psi(r, \varepsilon, \Omega) = \rho_{in}(r) \int_{\varepsilon}^{\infty} \int_{S^2} \sigma_{in}(\varepsilon', \varepsilon, \Omega' \cdot \Omega) \psi(r, \varepsilon', \Omega') d\Omega' d\varepsilon' + \\
+ \rho_{el}(r) \int_{S^2} \sigma_{el}(r, \varepsilon, \Omega' \cdot \Omega) \psi(r, \varepsilon, \Omega') d\Omega' - \rho_{tot}(r) \sigma_{tot}(\varepsilon) \psi(r, \varepsilon, \Omega).
\]

We can define the M1 model as:

\[
\nabla_x \psi^{(1)} = \frac{\partial}{\partial \varepsilon} (S \psi^{(0)}) \\
\nabla_x \psi^{(2)} = -(T_{M\text{oller}} + T_{M\text{ott}}) \psi^{(1)} + \frac{\partial}{\partial \varepsilon} (S \psi^{(1)})
\]

In this way we have defined the moments of the LBT equation. Now the system has three variables and two equations. The system can be closed only writing the second moment in term of the first and zero moments.
The entropy maximization

To close the system we determine a distribution function that maximizes the entropy using the Boltzmann’s H-theorem:

\[
\psi_{M_1} = \max_{\psi > 0} \left\{ -\int \psi(r, \epsilon, \Omega) \log \psi(r, \epsilon, \Omega) \, d\Omega \right\}, \quad \begin{cases} 
\psi^{(0)}(r, \epsilon) = \int_{S^2} \psi(r, \epsilon, \Omega) \, d\Omega \\
\psi^{(1)}(r, \epsilon) = \int_{S^2} \Omega \psi(r, \epsilon, \Omega) \, d\Omega 
\end{cases}
\]

The function that maximizes the Boltzmann’s H-theorem is an exponential:

\[
\psi_{ME} = a_0 e^{-\Omega \cdot a_1}
\]

where \(a_0 \geq 0\) is a scalar and \(a_1\) is a vector in \(\mathbb{R}^3\).

The distribution function is:

- subject to restriction of its degrees of freedom
- generated by a non negative underlying distribution function
M₁ deterministic model
The closure of the system

Now we can write the second moment in term of the zeroth and first moments as:

\[
\psi^{(2)}(r, \epsilon) = \int_{S^2} (\Omega \otimes \Omega) \cdot \psi(r, \epsilon, \Omega) \, d\Omega = \\
= \psi^{(0)} \left( \frac{1 - \chi(\alpha)}{2} I + \frac{3\chi(\alpha) - 1}{2} \cdot \left( \frac{\psi^{(1)}}{|\psi^{(1)}|} \otimes \frac{\psi^{(1)}}{|\psi^{(1)}|} \right) \right)
\]

Where:

\[
\alpha = \frac{\psi^{(1)}}{\psi^{(0)}} = \frac{1 - |a_1| \coth(|a_1|)}{|a_1|^2} a_1, \quad \chi = \frac{|a_1|^2 - 2|a_1| \coth(|a_1|) + 2}{|a_1|^2}
\]

Finally we can calculate the dose distribution as:

\[
D(r) = \frac{T}{\rho(r)} \int_{0}^{\infty} S(r, \epsilon')\psi^{(0)}(r, \epsilon') \, d\epsilon'
\]

To calculate the dose only the zeroth moment is needed!
The transport, attenuation and scattering, are based on microscopic (physics) cross sections in the energy range from 1 keV up to 100 MeV.

- All particles, photons, electrons, positrons, primary, scattered or secondary are treated simultaneously and computations are performed in 1D, 2D or 3D.

The M1 code, differently from other algorithms as Pencil Beam, AAA or CCC, calculates the cross sections for each energy group.
Electron external beam
The validation scheme

Perform experimental measurements of deposited dose in homogeneous target;

Perform simulations to predict the profiles of dose at different depths in inhomogeneous phantom;

Find a source spectrum that can match the deposited dose curve in M1 and MCNPX;

Perform experimental measurements to validate the predicted profiles.
Electron external beam
Dose profile in presence of inhomogeneity and at different energies.

The M1 code can reach the same precision of a Monte Carlo also in complex cases where the pencil-beam algorithm fails.
Even in case of strong density gradient, as in this case, M1 can retrieve physical effects that are prerogative only of MC codes.
Photon external beam
Gaussian shape spectrum: 100 keV and 500 keV

Also in case of photon propagation M1 retrieves the MC code and calculates the contribution of the backscattered electron at the interface.
The M1 model can achieve the dose distribution calculated by the PENELOPE code.

Photon external beam
Gaussian shape spectrum centered at 5 MeV

The M1 model can achieve the dose distribution calculated by PENELOPE code.

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Brachytherapy
Introduction

The brachytherapy dosimetry is based on simplistic assumptions *

✦ The dose distribution is calculated in homogeneous liquid water phantom considering cylindrically symmetric sources;
✦ Inter-seed attenuation and inter-applicator shielding are not treated by current clinical calculations;
✦ The real chemical composition of the different tissues is not taken into account;
✦ The effects of patient dimension are not considered and may modify the dose distributions at the interfaces.

This approach is fast and practical in the clinical context but can lead to inaccuracies **.

Isodose in inhomogeneous water phantom - E = 30 keV

Overestimation of the dose in the homogeneous phantom due to the missing scatter effect from tissue-air interface.

The deterministic M1 results are not noisy as the statistic MC ones.
Heterogeneities as bone insert or air cavity affect scatter conditions and the isodose contours. Nevertheless, the differences remain small and only low dose lines are altered.
Brachytherapy

Isodose in a breast numerical phantom - $E = 100$ keV

The phantom induces dose build-up, density heterogeneities and finite patient effects.

The M1 model catches all features but shows some discrepancies as an overestimation of the dose in the mathematical lung.
Conclusions and on going work

Conclusions:

- M1 code can calculate the dose distributions with high precision in agreement with MC codes;
- M1 in term of ratio precision/calculation time should be better than any other deterministic code developed so far;
- M1 is structurally suitable both for external and for internal radiotherapy.

On going work:

- The code still has large margin to optimization;
- The physics will be completed and improved in order to take into account different materials;
- In next months we will collaborate with the IUCT - Oncopole for the photon validation and direct comparison with Acuros®.
Thanks for your attention!